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ON THE MOTION OF ELECTRONS IN AN ALTERNATING ELECTRIC FIELD

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Zusammenfassung

In der Einleitung wird gezeigt dass in einer Hochvakuum-Elektronenröhre, bestehend aus Kathode, zwei Gittern und Anode, durch Anlegen geeigneter Gleich- und Wechselspannungen hoher Frequenz erreicht werden kann, dass einige Elektronen eine grössere kinetische Energie erhalten als mit dem Augenblickswert des Potentialfeldes an der Stelle, wo sie sich befinden, korrespondiert. In Abschnitt 2 wird die kinetische Energie der Elektronen aus den Bewegungsgleichungen abgeleitet. Der gewonnene Ausdruck wird in Abschnitt 3 diskutiert und auf Sonderfälle angewandt und in Abschnitt 4 mit Messungen verglichen, wobei sich eine genügende Übereinstimmung ergibt. Als Anwendung wird eine neue Art der Gleichrichtung einer Wechselspannung hoher Frequenz beschrieben.

I. *Introduction.* We have met some problems connected with modern multigrid vacuum tubes, in which it is necessary to consider the motion of electrons in alternating electric fields. As a simple case of this kind, we take the configuration of fig. 1. A constant negative potential V_a is given to the anode with respect to the cathode and furthermore a sinusoidal voltage $E_a \cos \omega t$, the peak value E_a of which is smaller than the negative potential V_a. Grid nr. 2 is given a constant positive potential V_2 (of the order of 100 Volts) and grid 1 a constant negative potential V_1 (e.g. —2 V) with respect to the cathode. The field strength F between grid 2 and anode is given by

$$F = \left\{\frac{V_2 - V_a}{d} - \frac{E_a}{d} \cos \omega t\right\}$$

where d is the distance from grid nr. 2 to the anode. The path of electrons, drawn from the cathode through the grid nr. 2 in the grid nr. 2 — anode — space (ga-space) is illustrated by fig. 1.

We shall first show, that some of these electrons can leave the ga-space with a greater kinetic energy than corresponds to the moment of their entrance into this space. We select for an example an electron, which enters the ga-space at a moment, shown by fig. 2 relative to the alternating anode voltage. The points *A*, *B*, *C* in the ga-space of fig. 1 correspond with the moments *A*, *B*, *C* of the time scale on fig. 2. At the moment *A* and at all times between *A* and *B* (fig. 2) the negative potential of the anode is less than would be the case, if no alternating voltage were given to the anode.

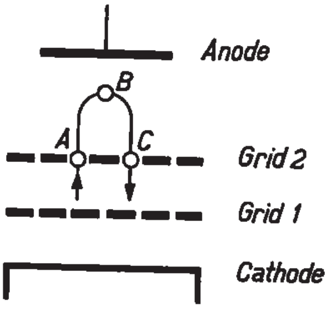

Fig. 1. Illustration of vacuum tube under discussion. Grid nr. 1 has a negative potential with respect to the cathode, grid nr. 2 a positive potential, the anode a negative potential and a superimposed alternating voltage, the peak value of which is smaller than the absolute value of the anode potential. An electron enters the grid nr. 2 — anode — space at *A*, is slowed down and returns at *B* and leaves this space at *C*.

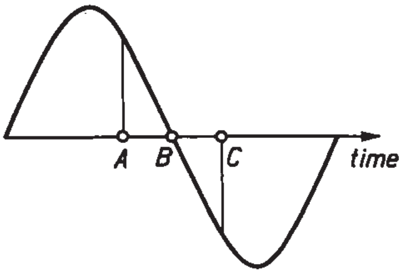

Fig. 2. Alternating anode voltage (vertical) as a function of time (horizontal). The points *A*, *B* and *C* of fig. 1 are shown in their relative position to the anode voltage.

The selected electron will hence be less slowed down on the way from *A* to *B* in fig. 1 than would be the case without alternating voltage on the anode. The point *B* in fig. 1 will lie nearer to the anode with than without alternating voltage for this selected electron. At all moments between *B* and *C* (fig. 2) the anode potential is more negative than without alternating voltage on the anode. Hence the selected electron will be more accelerated during a longer way (fig. 1) than without alternating voltage. We conclude that this selected electron will leave the ga-space at *C* (fig. 1) with a greater kinetic energy than it possessed at the point *A*. Of course, if for an electron the points *A*, *B*, *C* of fig. 1 are situated anywhere on the downward slope of the sine-curve in fig. 2, also a gain of energy results. Electrons that have gained energy in the ga-space can land on the negative grid nr. 1 and may be measured as a current to this grid.

It is, of course, equally possible to select electrons with an entrance time into the ga-space relative to the alternating anode voltage, thus, that their energy at the point of leaving the ga-space is less than at the entrance point into that space.

II. *Equations of motion.* The equation of motion for an electron, entering the ga-space at $x = 0$ (A in fig. 1) at a time $t = 0$, neglecting space charge, is:

$$\frac{d^2x}{dt^2} = -\frac{e}{m}\left\{\frac{V_2 - V_a}{d} - \frac{E_a}{d}\cos(\omega t + \varphi),\right\} \tag{1}$$

where $V_a + E_a \cos\varphi$ is the anode voltage at the time $t = 0$.

At the moment $t = 0$ we have $x = 0$ and $dx/dt = v_0 = (2e\,V_2/m)^{\frac{1}{2}}$. Eq. (1) yields on integrating it twice:

$$x = v_0 t - \frac{2v_0}{t_0}\left[\frac{t^2}{2} + \frac{p}{\omega^2}\{\cos(\omega t + \varphi) + \omega t \sin\varphi - \cos\varphi\}\right] \tag{2}$$

with

$$t_0 = 2\,dmv_0/e(V_2 - V_a) \text{ and } p = E_a/(V_2 - V_a),$$

the time t_0 being the electron transit time from A to C (fig. 1) if $E_a = 0$ (no alternating voltage on the anode).

If we take $x = 0$, we get two solutions, one is trivial: $t = 0$ and the other marks the point C of fig. 1, say $t = T$. T may be developed into a power series of p:

$$T = t_0 + C_1 p + C_2 p^2 + C_3 p^3 + \ldots\ldots, \tag{3}$$

converging rapidly for small values of p. In our case, neglecting p^3 leads to a sufficient approximation. The constants C_1 and C_2 are given by:

$$C_1 = -\frac{2}{\omega^2 t_0}\{(\omega t_0 - \sin\omega t_0)\sin\varphi - (1 - \cos\omega t_0)\cos\varphi\},$$

$$C_2 = -\frac{C_1^2}{t_0} - \frac{2C_1}{\omega t_0}\{(1 - \cos\omega t_0)\sin\varphi - \sin\omega t_0\cos\varphi\}.$$

Differentiating eq. (2), inserting the value $t = T$ and calculating dx/dt for $t = T$ yields the electron velocity v_1 at the point where it leaves the ga-space. By taking $[mv_1^2/2 - mv_0^2/2]$ we get the gain (or loss) of energy of this electron in the ga-space.

This may be expressed in electronvolts V_e:

$$V_e = pV_2\,\Phi \sin(\varphi + \omega t_0/2) + p^2V_2\,\Phi \sin(\omega t_0/2) + \\ + 4p^2V_2 \frac{\omega t_0 - \sin \omega t_0}{(\omega t_0)^2} \sin(2\varphi + \omega t_0) + \ldots\ldots \qquad (4)$$

The function Φ of $(\omega t_0/2)$ is given by:

$$\Phi = 4\,\{\sin(\omega t_0/2) - (\omega t_0/2)\cos(\omega t_0/2)\}\,(\omega t_0/2)^{-2} \qquad (5)$$

The functions Φ and $\Phi \sin(\omega t_0/2)$ are shown in fig. 3.

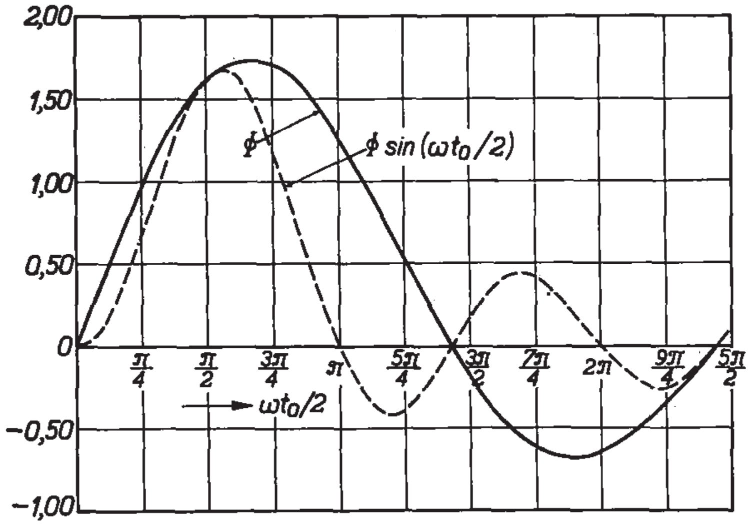

Fig. 3. The functions Φ and $\Phi \sin(\omega t_n/2)$ of eq. (4) as functions of $(\omega t_0/2)$.

III. *Discussion of the equation for the energy gain (or loss) of th electrons.* The time t_0 in eq. (4) is the electron transit time from A to C (fig. 1) if $E_a = 0$ (no alternating voltage on the anode). I considering the gain of energy it is more convenient to use $\pi t_0/$ instead of $\omega t_0/2$, where τ is the period of the alternating voltage. W may consider the modulus of V_e : $/V_e/$ as a good measure for th obtainable gain of energy. For small values of $p = E_a/(V_2 - V_a)$, i.e considering only the first term of eq. (4), a maximum gain of energ of about 1,7 times the amplitude E_a of the alternating anode voltag is obtained for t_0/τ somewhat greater than $\frac{1}{2}$. If the frequency i further increased, t_0/τ increases too, whereas $/V_e/$ decreases an becomes very small for t_0/τ about 3/2. Greater values of t_0/τ correspond again with more gain of energy, a second maximum of abou 0,7 E_a is obtained for $t_0/\tau = 2$, etc. (see fig. 3).

In applications to practical vacuum tubes, t_0/τ may often b considered as a small quantity. If $p \ll 1$ also, we have, by eq. (4):

$$V_e = \tfrac{2}{3}\omega\, E_a \frac{d}{(2eV_2/m)^{\frac{1}{2}}}\left(\frac{V_2}{V_2 - V_a}\right)^2 \sin\varphi \tag{6}$$

Hence, the gain of energy is, in this case, proportional to the distance d between grid nr. 2 and the anode and proportional to the frequency. By eq. (6) it is possible to choose the data of a vacuum tube either favourable or unfavourable for this effect.

We now consider many electrons, entering the ga-space at times, corresponding to different values of φ.

By integrating eq. (4) over φ, it is seen that the term proportional to p, i.e. the first order effect, becomes zero. This means, that to a first approximation as many electrons lose energy as gain energy in the ga-space. Some electrons gain energy and this gain of energy is supplied by the alternating voltage source. Other electrons, however, lose energy in the ga-space and this energy is delivered to the anode voltage source. Thus, in the mean, these amounts balance out to a first approximation.

If we take the terms of eq. (4), which are proportional to p^2, into consideration, and again integrate over φ, a positive mean supply of energy by the alternating voltage source results, which is due to the second term of eq. (4), as this does not contain φ. This mean supply of energy by the voltage source to the valve may be expressed by a positive real admittance $1/R_a$ between anode and cathode. The supplied energy is then equal to $E_a^2/2R_a$. If we assume, that the current, entering the ga-space is I_0, the number of electrons, entering this space per second, is I_0/e. As the energy gain for any electron, taking mean values over long time-intervals, is $e\overline{V}_e$, we have:

$$\frac{E_a^2}{2R_a} = e\overline{V}_e\, I_0/e = \frac{I_0 E_a^2}{(V_2 - V_a)^2}\, V_2\, \Phi \sin(\omega t_0/2)$$

or

$$\frac{1}{R_a} = 2\frac{I_0}{(V_2 - V_a)^2}\, V_2\, \Phi \sin(\omega t_0/2) \tag{7}$$

The function $\Phi \sin(\omega t_0/2)$ is shown in fig. 3. If we take $\omega t_0 \ll 1$, as is often the case in practical measurements, we obtain:

$$\frac{1}{R_a} = \frac{16}{3}\frac{m}{e}\frac{\omega^2 I_0 d^2}{V_2^2}\left(\frac{V_2}{V_2 - V_a}\right)^4 \tag{8}$$

This expression was derived previously by Messrs. Bakker and De Vries (see reference nr. 1). Inserting the value $e/m =$

$= 17{,}6.10^{14}$ (cm^2 sec^{-2} Volt^{-1}), and taking I_0 in amperes, d in cm, V_2 and V_a in volts, R_a is obtained in Ohms.

In the above calculations only electrons which enter the ga-space one single time, are taken into account. The electrons, which leave this space, are partly absorbed by grid nr. 2, by grid nr. 1 (only a few ones) or by the cathode. The other ones are returned to the ga-space and run through this space a second time. This process is repeated many times by some electrons. Among these, there will be a few ones that have gained energy during their first parcourse in the ga-space and that will gain energy also during their second, third, etc. parcourse of that space. Taking T_1 to be the time necessary for an average electron to return from the point C (fig. 1) to the point A on its way through the grid 1 — grid 2 — space, this process of accumulating energy may be followed up by considering eq. (4). Taking again $p \ll 1$ and neglecting the terms proportional to p^2, and taking $\varphi = 0$, $\omega t_0/2 \sim \pi/2$, the selected electron will gain a maximum amount of energy during its first parcourse. The second parcourse of the ga-space begins at the time $t = T + T_1$ and the electron will begin its second parcourse under the same favourable conditions as the first one, if $T + T_1 = 2\pi/\omega$. Hence, if this condition is realized, some electrons will be able to accumulate a considerable amount of energy by travelling several times through the ga-space. But, of course, only few electrons will gain this amount.

IV. *Experimental results.* We have in fig. 5 a valve circuit, which transforms an alternating voltage (on the anode) into a direct current (to grid nr. 1). An apparatus of this kind is generally called a rectifier.

Some measured results with a tetrode according to fig. 1 are given in fig. 4, whereas fig. 5 shows the circuit, which was used for measuring these values. The direct current I to grid nr. 1, caused by electrons which have gained energy in the ga-space, is directly transformed into volts in these measurements. They confirm the conclusion, drawn above from eq. (4) that the gain of energy, expressed in electronvolts, is of the same order as the amplitude of the alternating anode volts.

This special rectifier will work only if the electron transit time T in the ga-space is comparable with the period of the alternating voltage on the anode. We have, therefore, called it a *transit time rectifier.*

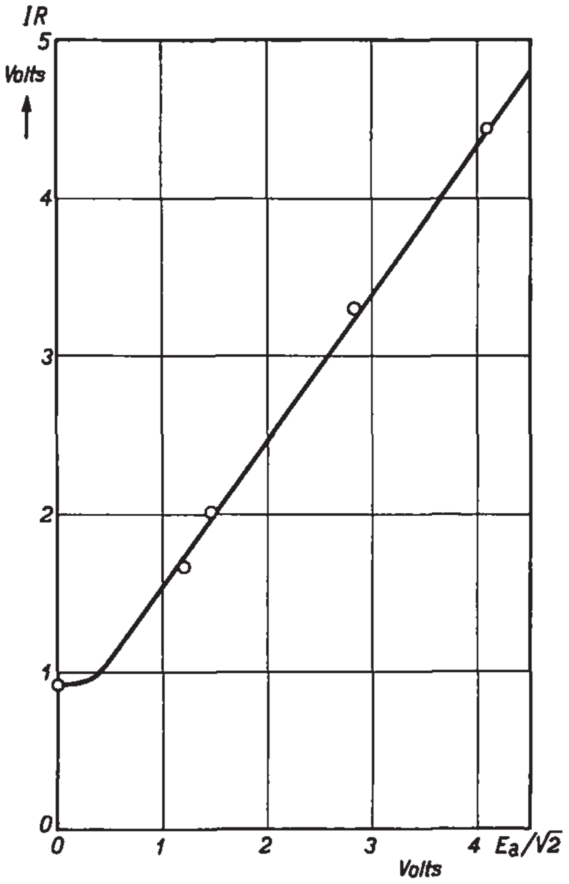

Fig. 4. Vertical: Product of d.c. I (see fig. 5) and resistance R (fig. 5). Horizontal: effective value of alternating anode voltage. For small values of E_a the product IR is proportional to E^2_a and for large values of E_a this product is proportional to E_a. This behaviour corresponds with the well known detection curve of a diode. Wavelength 10 m.

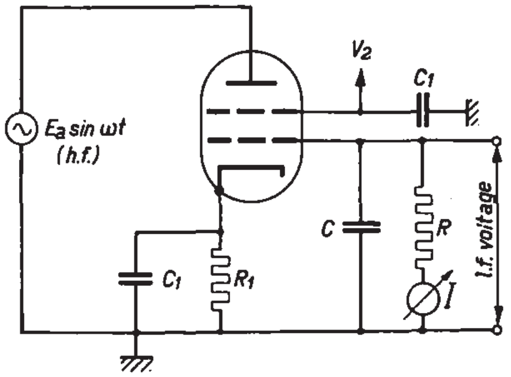

Fig. 5. Circuit used in obtaining the experimental values of fig. 4. The vacuum tube was a RCA type 35 tetrode. C_1 are block condensers of $10^4 \mu\mu F$, C a condenser of $100\ \mu\mu F$, R a resistance of 10^6 Ohm, R_1 a resistance of 300 Ohms. I is the current, measured in fig. 4, V_2 the grid nr. 2 potential and $E_a \sin \omega t$ the anode alternating voltage. The negative anode potential is due to the tube current through R_1 with an additional battery tension and the negative potential of grid nr. 1 is moreover due to the current I through R.

The direct current (transit time current) to grid nr. 1 flows in pulses. This is illustrated by fig. 6. The current pulses have a maximum corresponding with the downward variation (fig. 2) of the alternating voltage on the anode. The electron transit time causes a

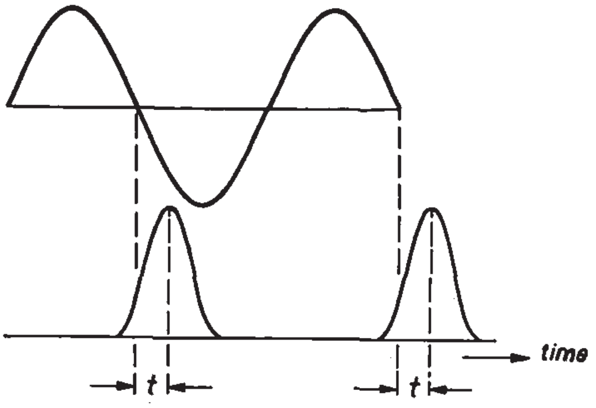

Fig. 6. Qualitative illustration of the current I to grid nr. 1 of fig. 5 (lower part), if R is omitted and the relative position to the anode alternating voltage (upper part).

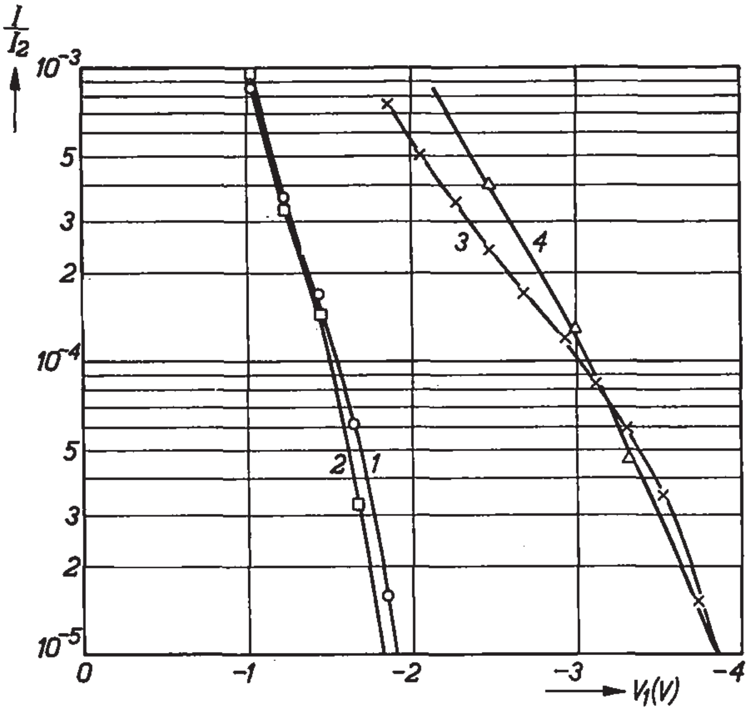

Fig. 7. Vertical: Current I of fig. 5 (R omitted) over the current I_2 to grid nr. 2 Horizontal: negative potential of grid nr. 1 (resistance R_1 of fig. 5 replaced by an adjustable battery). Curve nr .1 (circles): $V_2 = 50$ V, $V_a = -10$ V, $E_a = 1{,}4$ V; Curve nr. 2 (squares): $V_2 = 50$ V, $V_a = -50$ V, $E_a = 1{,}4\ (100/60)^2$ V; Curve nr. 3 (crosses): $V_2 = 100$ V, $V_a = -20$ V, $E_a = 2{,}8\ \sqrt{2}$ V; Curve nr. 4 (triangles): $V_2 = 50$ V, $V_a = -10$ V, $E_a = 2{,}8$ V. Wavelength for all curves : 10m.

certain retardation of the pulse maximum as compared with the zero point of the alternating anode voltage.

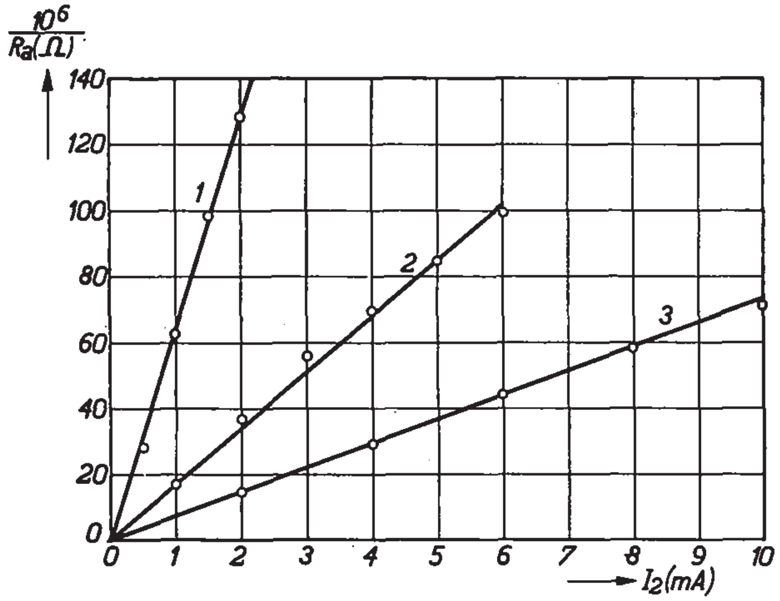

Fig. 8. Experimental verification of eq. (8). Vertical: $10^6/R_a$, where $1/R_a$ is the real part of the admittance between anode and cathode expressed in Ohm^{-1}. Horizontal: Current I_2 to grid nr. 2. Tube as in figs. 5 and 7. Wavelength of anode alternating voltage: 9,4 m. Circuit as in fig. 5 with R omitted. Curve 1: $V_2 = 30$ V, $V_a = —2$ V. Curve 2: $V_2 = 60$ V, $V_a = —4$ V. Curve 3: $V_2 = 90$ V, $V_a = —6$ V. Amplitude E_a of anode alternating voltage: some tenths of a volt.

In order to check eq. (6) further we have carried out some measurements of the current I to grid nr. 1 due to electrons, which have gained energy in the ga-space. The circuit of fig. 5 was used, omitting, however, the resistance R. The current to grid nr. 1 will be proportional to the total number of electrons in the valve, if we neglect all space charge effects. The current I_2 to grid nr. 2 will be a measure for the total number of electrons, partaking of the processes in the valve. Hence we plot I/I_2 as a function of the negative potential of grid nr. 1 and as a function of the alternating voltage E_a (fig. 7). The coincidence of curves 1 and 2 and of curves 3 and 4 of fig. 7 confirm the proportionality of the gain of energy to $E_a V_2^{3/2}/(V_2 — V_a)^2$, as expressed by eq. (6). At $I/I_2 = 10^{-5}$ the curves 3 and 4 correspond with a negative potential of grid nr. 1, equal to —3,9 V, whereas the curves 1 and 2 correspond with —1,95 V. This may be correlated to the fact resulting from eq. (6), that the energy gain in electron-volts is two times as large for the curves 3 and 4 than for the curves 1 and 2.

Some measurements of $1/R_a$ are shown in fig. 8. These values show that $1/R_a$ is proportional to the current I_2 to grid nr. 2, which is assumed to be proportional to I_0. Hence $1/R_a$ is proportional to I_0, as it should be by eq. (8). Considering the values of V_2 and V_a for the three curves of fig. 8, the values of $1/R_a$ should, by eq. (8), be proportional to 1 : 1/4 : 1/9 for these curves, at the same value of I_2. So they are, approximately. This fig. 8 may even be used to confirm eq. (8) quantitatively. In the valve measured, $d = 0{,}63$ cm, I_0 for $I_2 = 2$ mA is approximately 3,5 mA. At $V_2 = 30$ V, $V_a = -2$ V and for $\omega = 2{,}0.10^8$ the calculated value of $10^6/R_a$ is 126 and the measured value is 128. As the points on the three curves of fig. 8 confirm the eq. (8) by their relative situation, they may all be said to afford a good quantitative confirmation of eq. (8).

We have observed that in some cases no proportionality of $1/R_a$ to I_2 was obtained experimentally, e.g. if V_a has a large negative value. In these cases a further investigation is necessary. Perhaps the action of space charge, which is neglected in the above considerations, can afford an explanation of these experiments.

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